What does *height* really mean? Part I: Introduction

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ABSTRACT: This is the first paper in a four-part series considering the fundamental question, "what does the word height really mean?" National Geodetic Survey (NGS) is embarking on a height modernization program in which, in the future, it will not be necessary for NGS to create new or maintain old orthometric height benchmarks. In their stead, NGS will publish measured ellipsoid heights and computed Helmert orthometric heights for survey markers. Consequently, practicing surveyors will soon be confronted with coping with these changes and the differences between these types of height. Indeed, although "height'" is a commonly used word, an exact definition of it can be difficult to find. These articles will explore the various meanings of height as used in surveying and geodesy and present a precise definition that is based on the physics of gravitational potential, along with current best practices for using survey-grade GPS equipment for height measurement. Our goal is to review these basic concepts so that surveyors can avoid potential pitfalls that may be created by the new NGS height control era. The first paper reviews reference ellipsoids and mean sea level datums. The second paper reviews the physics of heights culminating in a simple development of the geoid and explains why mean sea level stations are not all at the same orthometric height. The third paper introduces geopotential numbers and dynamic heights, explains the correction needed to account for the non-parallelism of equipotential surfaces, and discusses how these corrections were used in NAVD 88. The fourth paper presents a review of current best practices for heights measured with GPS.

Preliminaries

The National Geodetic Survey (NGS) is responsible for the creation and maintenance of the United State's spatial reference framework. In order to address unmet spatial infrastructure issues, NGS has embarked on a height modernization program whose "… most desirable outcome is a unified national positioning system, comprised of consistent, accurate, and timely horizontal, vertical, and gravity control networks, joined and maintained by the Global Positioning System (GPS) and administered by the National Geodetic Survey" (National Geodetic Survey 1998). As a result of this program, NGS is working with partners to maintain the National Spatial Reference System (NSRS).

In the past, NGS performed high-accuracy surveys and established horizontal and/or vertical coordinates in the form of geodetic latitude and longitude and orthometric height. The National Geodetic Survey is responsible for the federal framework and is continually developing new tools and techniques using new technology to more effectively and efficiently establish this framework, i.e., GPS and Continually Operating Reference System (CORS). The agency is working with partners to transfer new technology so the local requirements can be performed by the private sector under the supervision of the NGS (National Geodetic Survey 1998).

Instead of building new benchmarks, NGS has implemented a nation-wide network of continuously operating global positioning system (GPS) reference stations known as the CORS, with the intent that CORS shall provide survey control in the future. Although GPS excels at providing horizontal coordinates, it cannot directly measure an orthometric height; GPS can only directly provide ellipsoid heights. However, surveyors and engineers seldom need ellipsoid heights, so NGS has created highly sophisticated, physics-based, mathematical software models of the Earth's gravity field (Milbert 1991; Milbert and Smith 1996; Smith and Milbert 1999; Smith and Roman 2001) that are used in conjunction with ellipsoid heights to infer Helmert orthometric heights (Helmert 1890). As a result, practicing surveyors, mappers, and engineers working in the United States may be working with mixtures of ellipsoid and orthometric heights. Indeed, to truly understand the

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output of all these height conversion programs, one must come to grips with heights in all their forms, including elevations, orthometric heights, ellipsoid heights, dynamic heights, and geopotential numbers.

According to the *Geodetic Glossary* (National Geodetic Survey 1986), **height** is defined as, "The distance, measured along a perpendicular, between a point and a reference surface, e.g., the height of an airplane above the ground." Although this definition seems to capture the intuition behind height very well, it has a (deliberate) ambiguity regarding the reference surface (datum) from which the measurement was made.

Heights fall broadly into two categories: those that employ the Earth's gravity field as their datum and those that employ a reference ellipsoid as their datum. Any height referenced to the Earth's gravity field can be called a "geopotential height," and heights referenced to a reference ellipsoid are called "ellipsoid heights." These heights are not directly interchangeable; they are referenced to different datums and, as will be explained in subsequence papers, in the absence of site-specific gravitation measurements there is no rigorous transformation between them. This is a situation analogous to that of the North American Datum of 1983 (NAD83) and the North American Datum of 1927 (NAD27)— two horizontal datums for which there is no rigorous transformation.

The definitions and relationships between elevations, orthometric heights, dynamic heights, geopotential numbers, and ellipsoid heights are not well understood by many practitioners. This is perhaps not too surprising, given the bewildering amount of jargon associated with heights. The NGS glossary contains 17 definitions with specializations for "elevation," and 23 definitions with specializations for "height," although nine of these refer to other (mostly elevation) definitions. It is the purpose of this series, then, to review these concepts with the hope that the reader will have a better and deeper understanding of what the word "height" really means.

The Series

The series consists of four papers that review vertical datums and the physics behind height measurements, compare the various types of heights, and evaluate the current best practices for deducing orthometric heights from GPS measurements. Throughout the series we will enumerate figures, tables, and equations with a Roman numeral indicating the paper in the series from which it came. For example, the third figure in the second paper will be numbered, "Figure II.3".

This first paper in the series is introductory. Its purpose is to explain why a series of this nature is relevant and timely, and to present a conceptual framework for the papers that follow. It contains a review of reference ellipsoids, mean sea level, and the U.S. national vertical datums.

The second paper is concerned with gravity. It presents a development of the Earth's gravity forces and potential fields, explaining why the force of gravity does not define level surfaces, whereas the potential field does. The deflection of the vertical, level surfaces, the geoid, plumb lines, and geopotential numbers are defined and explained.

It is well known that the deflection of the vertical causes loop misclosures for horizontal traverse surveys. What seems to be less well known is that there is a similar situation for orthometric heights. As will be discussed in the second paper, geoid undulations affect leveled heights such that, in the absence of orthometric corrections, the elevation of a station depends on the path taken to the station. This is one cause of differential leveling loop misclosure. The third paper in this series will explain the causes of this problem and how **dynamic heights** are the solution.

The fourth paper of the series is a discussion of height determination using GPS. GPS measurements that are intended to result in orthometric heights require a complicated set of datum transformations, changing ellipsoid heights to orthometric heights. Full understanding of this process and the consequences thereof requires knowledge of all the information put forth in this review. As was mentioned above, NGS will henceforth provide the surveying community with vertical control that was derived using these methods. Therefore, we feel that practicing surveyors can benefit from a series of articles whose purpose is to lay out the information needed to understand this process and to use the results correctly.

The current article proceeds as follows. The next section provides a review of ellipsoids as they are used in geodesy and mapping. Thereafter follows a review of mean sea level and orthometric heights, which leads to a discussion of the national vertical datums of the United States. We conclude with a summary.

Reference Ellipsoids

A **reference ellipsoid**, also called **spheroid**, is a simple mathematical model of the Earth's shape. Although low-accuracy mapping situations might be able to use a spherical model for the Earth, when more accuracy is needed, a spherical model is inadequate, and the next more complex Euclidean shape is an ellipsoid of revolution. An ellipsoid of revolution, or simply an "ellipsoid," is the shape that results from rotating an ellipse about one of its axes. Oblate ellipsoids are used for geodetic purposes because the Earth's polar axis is shorter than its equatorial axis.

Local Reference Ellipsoids

Datums and cartographic coordinate systems depend on a mathematical model of the Earth's shape upon which to perform trigonometric computations to calculate the coordinates of places on the Earth and in order to transform between geocentric, geodetic, and mapping coordinates. The transformation between geodetic and cartographic coordinates requires knowledge of the ellipsoid being used, e.g., see (Bugayevskiy and Snyder 1995; Qihe et al. 2000; Snyder 1987). Likewise, the transformation from geodetic to geocentric Cartesian coordinates is accomplished by Helmert's projection, which also depends on an ellipsoid (Heiskanen and Moritz 1967, pp. 181-184) as does the inverse relationship; see Meyer (2002) for a review. Additionally, as mentioned above, measurements taken with chains and transits must be reduced to a common surface for geodetic surveying, and a reference ellipsoid provides that surface. Therefore, all scientifically meaningful geodetic horizontal datums depend on the availability of a suitable reference ellipsoid.

Until recently, the shape and size of reference ellipsoids were established from extensive, continental-sized triangulation networks (Gore 1889; Crandall 1914; Shalowitz 1938; Schwarz 1989; Dracup 1995; Keay 2000), although there were at least two different methods used to finally arrive at an ellipsoid (the "arc" method for Airy 1830, Everest 1830, Bessel 1841 and Clarke 1866; and the "area" method for Hayford 1909). The lengths of (at least) one starting and ending baseline were measured with instruments such as rods, chains, wires, or tapes, and the lengths of the edges of the triangles were subsequently propagated through the network mathematically by triangulation.

For early triangulation networks, vertical distances were used for reductions and typically came from trigonometric heighting or barometric measurements although, for NAD 27, "a line of precise levels following the route of the triangulation was begun in 1878 at the Chesapeake Bay and reached San Francisco in 1907" (Dracup 1995). The ellipsoids deduced from triangulation networks were, therefore, custom-fit to the locale in which the survey took place. The result of this was that each region in the world thus measured had its own ellipsoid, and this gave rise to a large number of them; see NIMA WGS 84 Update Committee (1997) and Meyer (2002) for a review and the parameters of many ellipsoids. It was impossible to create a single, globally applicable reference ellipsoid with triangulation networks due to the inability to observe stations separated by large bodies of water.

Local ellipsoids did not provide a vertical datum in the ordinary sense, nor were they used as such. Ellipsoid heights are defined to be the distance from the surface of the ellipsoid to a point of interest in the direction normal to the ellipsoid, reckoned positive away from the center of the ellipsoid. Although this definition is mathematically well defined, it was, in practice, difficult to realize for several reasons. Before GPS, all high-accuracy heights were measured with some form of leveling, and determining an ellipsoid height from an orthometric height requires knowledge of the deflection of the vertical, which is obtained through gravity and astronomical measurements (Heiskanen and Moritz 1967, pp. 82-84).

Deflections of the vertical, or high-accuracy estimations thereof, were not widely available prior to the advent of high-accuracy geoid models. Second, the location of a local ellipsoid was arbitrary in the sense that the center of the ellipsoid need not coincide with the center of the Earth (geometric or center of mass), so local ellipsoids did not necessarily conform to mean sea level in any obvious way. For example, the center of the Clarke 1866 ellipsoid as employed in the NAD 27 datum is now known to be approximately 236 meters from the center of the Global Reference System 1980 (GRS 80) as placed by the NAD83 datum. Consequently, ellipsoid heights reckoned from local ellipsoids had no obvious relationship to gravity. This leads to the ever-present conundrum that, in certain places, water flows "uphill," as reckoned with ellipsoid heights (and this is still true even with geocentric ellipsoids, as will be discussed below). Even so, some local datums (e.g., NAD 27, Puerto Rico) were designed to be "best fitting" to the local geoid to minimize geoid heights, so in a sense they were "fit" to mean sea level. For example, in computing plane coordinates on NAD 27, the reduction of distances to the ellipsoid was called the "Sea Level Correction Factor"!

In summary, local ellipsoids are essentially mathematical fictions that enable the conversion between geocentric, geodetic, and cartographic coordinate systems in a rigorous way and, thus, provide part of the foundation of horizontal geodetic datums, but nothing more. As reported by Fischer (2004), "O'Keefe[1] tried to explain to me that conventional geodesy used the ellipsoid only as a mathematical computation device, a set of tables to be consulted during processing, without the slightest thought of a third dimension."

Equipotential Ellipsoids

In contrast to local ellipsoids that were the product of triangulation networks, globally applicable reference ellipsoids have been created using very long baseline interferometry (VLBI) for GRS 80 (Moritz 2000), satellite geodesy for the World Geodetic System 1984 (WGS 84) (NIMA WGS 84 Update Committee 1997), along with various astronomical and gravitational measurements. Very long baseline interferometry and satellite geodesy permit high-accuracy baseline measurement between stations separated by oceans. Consequently, these ellipsoids model the Earth globally; they are not fitted to a particular local region. Both WGS 84 and GRS 80 have size and shape such that they are a best-fit model of the geoid in a least-squares sense. Quoting Moritz (2000, p.128),

> The Geodetic Reference System 1980 has been adopted at the XVII General Assembly of the IUGG in Canberra, December 1979, by means of the following: … recognizing that the Geodetic Reference System 1967 … no longer represents the size, shape, and gravity field of the Earth to an accuracy adequate for many geodetic, geophysical, astronomical and hydrographic applications and considering that more appropriate values are now available, recommends … that the Geodetic Reference System 1967 be replaced by a new Geodetic Reference System 1980, also based on the theory of the geocentric equipotential ellipsoid, defined by the following constants:
>
> - Equatorial radius of the Earth: $a = 6378137$ m;
> - Geocentric gravitational constant of the Earth (including the atmosphere): $GM = 3{,}986{,}005 \times 10^8 \text{ m}^3 \text{ s}^{-2}$;
> - Dynamical form factor of the Earth, excluding the permanent tidal deformation: $J_2 = 108{,}263 \times 10^{-8}$; and
> - Angular velocity of the Earth: $\omega = 7292115 \times 10^{-11} \text{ rad s}^{-1}$.

Clearly, equipotential ellipsoid models of the Earth constitute a significant logical departure from local ellipsoids. Local ellipsoids are purely geometric, whereas equipotential ellipsoids include the geometric but also concern gravity. Indeed, GRS 80 is called an "equipotential ellipsoid" (Moritz 2000) and, using equipotential theory together with the defining constants listed above, one *derives* the flattening of the ellipsoid rather than measuring it geometrically. In addition to the logical departure, datums that employ GRS 80 and WGS 84 (e.g., NAD 83, ITRS, and WGS 84) are intended to be geocentric, meaning that they intend to place the center of their ellipsoid at the Earth's center of gravity. It is important to note, however, that NAD 83 currently places the center of GRS 80 roughly two meters away from the center of ITRS and that WGS 84 is currently essentially identical to ITRS.

Equipotential ellipsoids are both models of the Earth's shape and first-order models of its gravity field. Somiglinana (1929) developed the first rigorous formula for normal gravity (also, see Heiskanen and Moritz (1967, p. 70, eq. 2-78)) and the first internationally accepted equipotential ellipsoid was established in 1930. It had the form:

$$g_0 = 9.78046(1 + 0.0052884\sin^2\varphi - 0.0000059\sin^2 2\varphi) \qquad \text{(I.1)}$$

where:

g_0 = acceleration due to gravity at a distance 6,378,137 m from the center of the idealized Earth; and

φ = geodetic latitude (Blakely 1995, p.135).

The value g_0 is called **theoretical gravity** or **normal gravity**. The dependence of this formula on geodetic latitude will have consequences when closure errors arise in long leveling lines that run mostly north-south compared to those that run mostly east-west. The most modern reference ellipsoids are GRS 80 and WGS 84. As given by Blakely (1995, p.136), the closed-form formula for WGS 84 normal gravity is:

[illegible] (I.2)

Figure I.1 shows a plot of the difference between Equation I.1 and Equation I.2. The older model has a larger value throughout and has, in the worst case, a magnitude greater by 0.000163229 m/s^2 (i.e., about 16 mgals) at the equator.

Equipotential Ellipsoids as Vertical Datums

Concerning the topic of this paper, perhaps the most important consequence of the differences between local and equipotential ellipsoids is that

[1] John O'Keefe was the head of geodetic research at the Army Map Service.

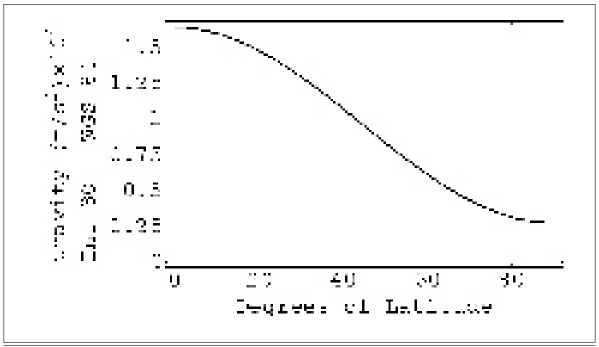

Figure I.1. The difference in normal gravity between the 1930 International Gravity Formula and WGS 84. Note that the values on the abscissa are given 10,000 times the actual difference for clarity.

equipotential ellipsoids are more suitable to be used as vertical datums in the ordinary sense than local ellipsoids and, in fact, they are used as such. In particular, GPS-derived coordinates expressed as geodetic latitude and longitude present the third dimension as an ellipsoid height. This constitutes a dramatic change from the past. Before, ellipsoid heights were essentially unheard of, basically only of interest and of use to geodesists for computational purposes. Now, anyone using a GPS is deriving ellipsoid heights.

Equipotential ellipsoids are models of the gravity that would result from a highly idealized model of the Earth; one whose mass is distributed homogeneously but includes the Earth's oblate shape, and spinning like the Earth. The geoid is not a simple surface compared to an equipotential ellipsoid, which can be completely described by just the four parameters listed above. The geoid's shape is strongly influenced by the topographic surface of the Earth. As seen in Figure I.2, the geoid appears to be "bumpy," with apparent mountains, canyons, and valleys. This is, in fact, not so. The geoid is a convex surface by virtue of satisfying the Laplace equation, and its apparent concavity is a consequence of how the geoid is portrayed on a flat surface (Vanicek and Krakiwsky 1996). Figure I.2 is a portrayal of the ellipsoid height of the geoid as estimated by GEOID 03 (Roman et al. 2004). That is to say, the heights shown in the figure are the distances from GRS 80 as located by NAD 83 to the geoid; the ellipsoid height of the geoid. Such heights (the ellipsoid height of a place on the geoid) are called **geoid heights**. Thus, Figure. I.2 is a picture of geoid heights.

Even though equipotential ellipsoids are useful as vertical datums, they are usually unsuitable as a surrogate for the geoid when measuring orthometric heights. Equipotential ellipsoids are "best-fit" over the entire Earth and, consequently, they typically do not match the geoid particularly well in any specific place. For example, as shown in Figure I.2, GRS 80 as placed by NAD 83 is everywhere higher than the geoid across the conterminous United States; not half above and half below. Furthermore, as described above, equipotential ellipsoids lack the small-scale details of the geoid. And, like local ellipsoids, ellipsoid heights reckoned from equipotential ellipsoids also suffer from the phenomenon that there are places where water apparently flows "uphill," although perhaps not as badly as some local ellipsoids. Therefore, surveyors using GPS to determine heights would seldom want to use ellipsoid heights. In most cases, surveyors need to somehow deduce an orthometric height from an ellipsoid height, which will be discussed in the following papers.

Mean Sea Level

One of the ultimate goals of this series is to present a sufficiently complete presentation of orthometric heights that the following definition will be clear. In the *NGS glossary*, the term **orthometric height** is referred to **elevation, orthometric**, which is defined as, "The distance between the geoid and a point measured along the **plumb line** and taken positive upward from the geoid." For contrast, we quote from the first definition for **elevation**:

> The distance of a point above a specified surface of constant **potential**; the distance is measured along the direction of gravity between the point and the surface. #
>
> The surface usually specified is the geoid or an approximation thereto. Mean sea level was long considered a satisfactory approximation to the geoid and therefore suitable for use as a reference surface. It is now known that mean sea level can differ from the geoid by up to a meter or more, but the exact difference is difficult to determine.
>
> The terms **height** and **level** are frequently used as synonyms for elevation. In geodesy, height also refers to the distance above an ellipsoid…

It happens that lying within these two definitions is a remarkably complex situation primarily concerned with the Earth's gravity field and our attempts to make measurements using it as a frame of reference. The terms **geoid, plumb line, potential, mean sea level** have arisen, and they must be addressed before discussing orthometric heights.

For heights, the most common datum is mean sea level. Using mean sea level for a height datum is perfectly natural because most human activity occurs at or above sea level. However, creating a workable and repeatable mean sea level datum is somewhat subtle. The *NGS Glossary* definition of mean sea level is "The average location of the interface between ocean and atmosphere, over a period of time sufficiently long so that all random and periodic variations of short duration average to zero."

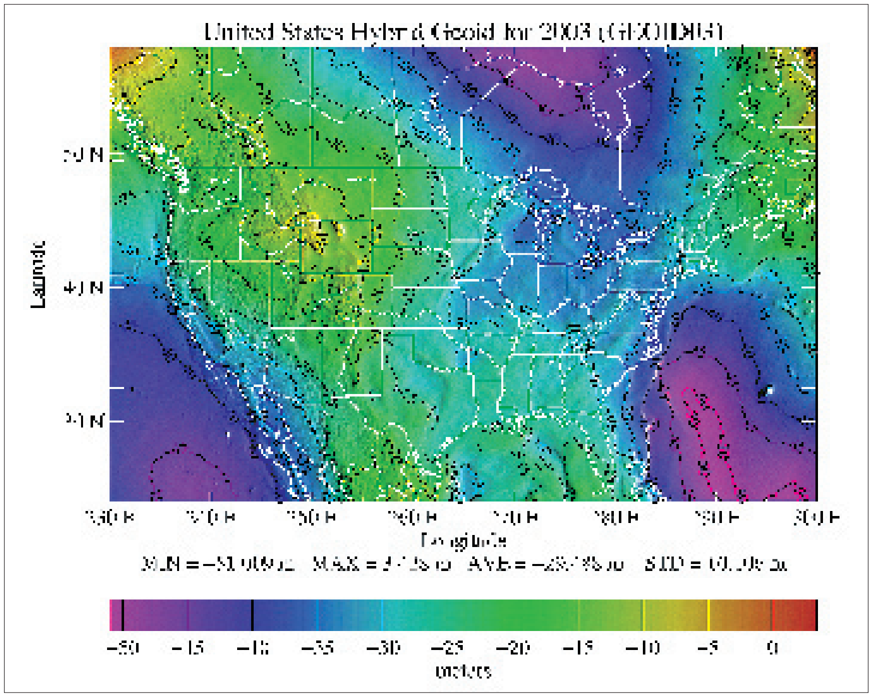

Figure I.2. Geoid heights with respect to NAD 83/GRS 80 over the continental United States as computed by GEOID03. [Source: http://www.ngs.noaa.gov/GEOID/GEOID03/images/geoid03.b.jpg.]

The National Oceanic and Atmospheric Administration's (NOAA) National Ocean Service (NOS) Center for Operational Oceanographic Products and Services (CO-OPS) has set 19 years as the period suitable for measurement of mean sea level at tide gauges (National Geodetic Survey 1986, p. 209). The choice of 19 years was chosen because it is the smallest integer number of years larger than the first major cycle of the moon's orbit around the Earth. This accounts for the largest of the periodic effects mentioned in the definition. See Bomford (1980, pp. 247-255) and Zilkoski (2001) for more details about mean sea level and tides. Local mean sea level is often measured using a tide gauge. Figure I.3 depicts a tide house, "a structure that houses instruments to measure and record the instantaneous water level inside the tide gauge and built at the edge of the body of water whose local mean level is to be determined."

It has been suspected at least since the time of the building of the Panama Canal that mean sea level might not be at the same height everywhere (McCullough 1978). The original canal, attempted by the French, was to be cut at sea level and there was concern that the Pacific Ocean might not be at the same height as the Atlantic, thereby causing a massive flood through the cut. This concern became irrelevant when the sea level approach was abandoned. However, the subject surfaced again in the creation of the National Geodetic Vertical Datum of 1929 (NGVD 29).

By this time it was a known fact that not all mean sea-level stations were the same height, a proposition that seems absurd on its face. To begin with, all mean sea-level stations are at an elevation of zero *by definition*. Second, water seeks its own level, and the oceans have no visible constraints preventing free flow between the stations (apart from the continents), so how could it be possible that mean sea level is not at the same height everywhere? The answer lies in differences in temperature, chemistry, ocean currents, and ocean eddies.

The water in the oceans is constantly moving at all depths. Seawater at different temperatures contains different amounts of salt and, consequently, has density gradients. These density gradients give rise to immense deep-ocean cataracts that constantly transport massive quantities of water from the poles to the tropics and back (Broecker 1983; Ingle 2000; Whitehead 1989). The sun's warming of surface waters causes the global-scale currents that are well-known to mariners in addition to other more subtle effects (Chelton et al. 2004). Geostrophic effects cause large-scale, persistent ocean eddies that push water against or away from the continents, depending on the direction of the eddy's circulation. These effects can create sea surface topographic variations of more than 50 centimeters (Srinivasan 2004). As described by Zilkoski (2001, p. 40) the differences are due to "… currents, prevailing winds and barometric pressures, water temperature and salinity differentials, topographic configuration of the bottom in the area of the gauge site, and other physical causes…"

In essence, these factors push the water and hold it upshore or away-from-shore further than

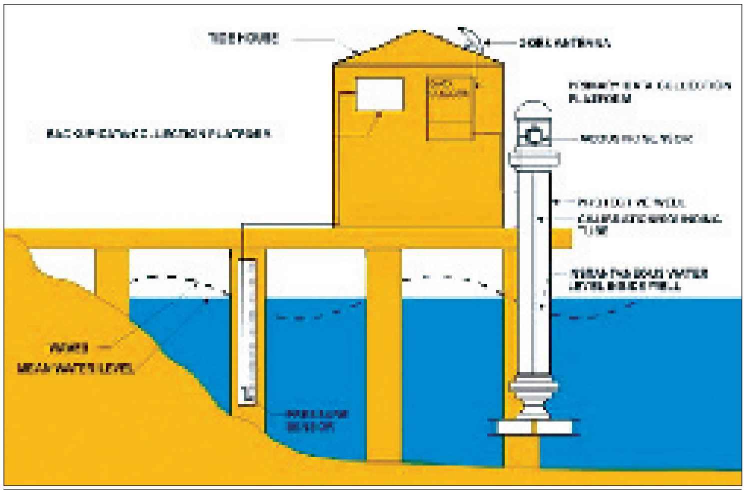

Figure I.3. The design of a NOAA tide house and tide gauge used for measuring mean sea level. (Source: http://oceanservice.noaa.gov/education/tides/media/supp_tide11a.html).

would be the case under the influence of gravity alone. Also, the persistent nature of these climatic factors prevents the elimination of their effect by averaging (e.g., see (Speed et al. 1996 a; Speed et al. 1996 b)). As will be discussed in more detail in the second paper, this gives rise to the seemingly paradoxical state that holding one sea-level station as a zero height reference and running levels to another station generally indicates that the other station is not also at zero height, even in the absence of experimental error and even if the two stations *are at the same gravitational potential*. Similarly, measuring the height of an inland benchmark using two level lines that start from different tide gauges generally results in two statistically different height measurements. These problems were addressed in different ways by the creation of two national vertical datums, NGVD 29 and North American Vertical Datum of 1988 (NAVD 88). We will now discuss the national vertical datums of the United States.

U.S. National Vertical Datums

The first leveling route in the United States considered to be of geodetic quality was established in 1856-57 under the direction of G.B. Vose of the U.S. Coast Survey, predecessor of the U.S. Coast and Geodetic Survey and, later, the National Ocean Service.[2] The leveling survey was needed to support current and tide studies in the New York Bay and Hudson River areas. The first leveling line officially designated as "geodesic leveling" by the Coast and Geodetic Survey followed an arc of triangulation along the 39th parallel. This 1887 survey began at benchmark A in Hagerstown, Maryland.

By 1900, the vertical control network had grown to 21,095 km of geodetic leveling. A reference surface was determined in 1900 by holding elevations referenced to local mean sea level (LMSL) fixed at five tide stations. Data from two other tide stations indirectly influenced the determination of the reference surface. Subsequent readjustments of the leveling network were performed by the Coast and Geodetic Survey in 1903, 1907, and 1912 (Berry 1976).

National Geodetic Vertical Datum of 1929 (NGVD 29)

The next general adjustment of the vertical control network, called the Sea Level Datum of 1929 and later renamed to the National Geodetic Vertical Datum of 1929 (NGVD 29), was accomplished in 1929. By then, the international nature of geodetic networks was well understood, and Canada provided data for its first-order vertical network to combine with the U.S. network. The two networks were connected at 24 locations through vertical control points (benchmarks) from Maine/New Brunswick to Washington/British Columbia. Although Canada did not adopt the "Sea Level Datum of 1929" determined by the United States, Canadian-U.S. cooperation in the general adjustment greatly strengthened the 1929 network. Table I.1 lists the kilometers of leveling involved in the readjustments and the number of tide stations used to establish the datums.

Year of Adjustment	Kilometers of Leveling	Number of Tide Stations
1900	21,095	5
1903	31,789	8
1907	38,359	8
1912	46,468	9
1929	75,159 (U.S.) 31,565 (Canada)	21 (U.S.) 5 (Canada)

Table I.1. Amount of leveling and number of tide stations involved in previous re-adjustments.

It was mentioned above that NGVD 29 was originally called the "Sea Level Datum of 1929." To eliminate some of the confusion caused by the original name, in 1976 the name of the datum was changed to "National Geodetic Vertical Datum

[2] This section consists of excerpts from Chapter 2 of Maune's (2001) *Vertical Datums*.

of 1929," eliminating all reference to "sea level" in the title. This was a change in name only; the mathematical and physical definitions of the datum established in 1929 were not changed in any way.

North American Vertical Datum of 1988 (NAVD 88)

The most recent general adjustment of the U.S. vertical control network, which is known as the North American Vertical Datum of 1988 (NAVD 88), was completed in June 1991 (Zilkoski et al. 1992). Approximately 625,000 km of leveling have been added to the NSRS since NGVD 29 was created. In the intervening years, discussions were held periodically to determine the proper time for the inevitable new general adjustment. In the early 1970s, the National Geodetic Survey conducted an extensive inventory of the vertical control network. The search identified thousands of benchmarks that had been destroyed, due primarily to post-World War II highway construction, as well as other causes. Many existing benchmarks were affected by crustal motion associated with earthquake activity, post-glacial rebound (uplift), and subsidence resulting from the withdrawal of underground liquids.

An important feature of the NAVD 88 program was the re-leveling of much of the first-order NGS vertical control network in the United States. The dynamic nature of the network requires a framework of newly observed height differences to obtain realistic, contemporary height values from the readjustment. To accomplish this, NGS identified 81,500 km (50,600 miles) for re-leveling. Replacement of disturbed and destroyed monuments preceded the actual leveling. This effort also included the establishment of stable "deep rod" benchmarks, which are now providing reference points for new GPS-derived orthometric height projects as well as for traditional leveling projects.

The general adjustment of NAVD 88 consisted of 709,000 unknowns (approximately 505,000 permanently monumented benchmarks and 204,000 temporary benchmarks) and approximately 1.2 million observations.

Analyses indicate that the overall differences for the conterminous United States between orthometric heights referred to NAVD 88 and NGVD 29 range from 40 cm to +150 cm. In Alaska the differences range from approximately +94 cm to +240 cm. However, in most "stable" areas, relative height changes between adjacent benchmarks appear to be less than 1 cm. In many areas, a single bias factor, describing the difference between NGVD 29 and NAVD 88, can be estimated and used for most mapping applications (NGS has developed a program called VERTCON to convert from NGVD 29 to NAVD 88 to support mapping applications). The overall differences between dynamic heights referred to International Great Lakes Datum of 1985 (IGLD 85) and IGLD 55 range from 1 cm to 37 cm.

International Great Lakes Datum of 1985 (IGLD 85)

For the general adjustment of NAVD 88 and the International Great Lakes Datum of 1985 (IGLD 85), a minimum constraint adjustment of Canadian–Mexican–U.S. leveling observations was performed. The height of the primary tidal benchmark at Father Point/Rimouski, Quebec, Canada (also used in the NGVD 1929 general adjustment), was held fixed as the constraint. Therefore, IGLD 85 and NAVD 88 are one and the same. Father Point/Rimouski is an IGLD water-level station located at the mouth of the St. Lawrence River and is the reference station used for IGLD 85. This constraint satisfied the requirements of shifting the datum vertically to minimize the impact of NAVD 88 on U.S. Geological Survey (USGS) mapping products, and it provides the datum point desired by the IGLD Coordinating Committee for IGLD 85. The only difference between IGLD 85 and NAVD 88 is that IGLD 85 benchmark values are given in dynamic height units, and NAVD 88 values are given in Helmert orthometric height units. Geopotential numbers for individual benchmarks are the same in both systems (the next two papers will explain dynamic heights, geopotential numbers, and Helmert orthometric heights).

Tidal Datums

Principal Tidal Datums

A vertical datum is called a tidal datum when it is defined by a certain phase of the tide. Tidal datums are local datums and are referenced to nearby monuments. Since a tidal datum is defined by a certain phase of the tide there are many different types of tidal datums. This section will discuss the principal tidal datums that are typically used by federal, state, and local government agencies: Mean Higher High Water (MHHW), Mean High Water (MHW), Mean Sea Level (MSL), Mean Low Water (MLW), and Mean Lower Low Water (MLLW).

A determination of the principal tidal datums in the United States is based on the average of observations over a 19-year period, e.g., 1988-2001. A specific 19-year Metonic cycle is denoted as a National Tidal Datum Epoch (NTDE). CO-OPS publishes the official United States local mean sea level values as defined by observations at the 175 station National Water Level Observation Network (NWLON). Users need to know which NTDE their data refer to.

- *Mean Higher High Water (MHHW):* MHHW is defined as the arithmetic mean of the higher high water heights of the tide observed over a specific 19-year Metonic cycle denoted as the NTDE. Only the higher high water of each pair of high waters of a tidal day is included in the mean. For stations with shorter series, a comparison of simultaneous observations is made with a primary control tide station in order to derive the equivalent of the 19-year value (Marmer 1951).
- *Mean High Water (MHW)* is defined as the arithmetic mean of the high water heights observed over a specific 19-year Metonic cycle. For stations with shorter series, a computation of simultaneous observations is made with a primary control station in order to derive the equivalent of a 19-year value (Marmer 1951).
- *Mean Sea Level (MSL)* is defined as the arithmetic mean of hourly heights observed over a specific 19-year Metonic cycle. Shorter series are specified in the name, such as monthly mean sea level or yearly mean sea level (e.g., Hicks 1985; Marmer 1951).
- *Mean Low Water (MLW)* is defined as the arithmetic mean of the low water heights observed over a specific 19-year Metonic cycle. For stations with shorter series, a comparison of simultaneous observations is made with a primary control tide station in order to derive the equivalent of a 19-year value (Marmer 1951).
- *Mean Lower Low Water (MLLW)* is defined as the arithmetic mean of the lower low water heights of the tide observed over a specific 19-year Metonic cycle. Only the lower low water of each pair of low waters of a tidal day is included in the mean. For stations with shorter series, a comparison of simultaneous observations is made with a primary control tide station in order to derive the equivalent of a 19-year value (Marmer 1951).

Other Tidal Values

Other tidal values typically computed include the Mean Tide Level (MTL), Diurnal Tide Level (DTL), Mean Range (Mn), Diurnal High Water Inequality (DHQ), Diurnal Low Water Inequality (DLQ), and Great Diurnal Range (Gt).

- *Mean Tide Level* (MTL) is a tidal datum which is the average of Mean High Water and Mean Low Water.
- *Diurnal Tide Level* (DTL) is a tidal datum which is the average of Mean Higher High Water and Mean Lower Low Water.
- *Mean Range* (Mn) is the difference between Mean High Water and Mean Low Water.
- *Diurnal High Water Inequality* (DHQ) is the difference between Mean Higher High Water and Mean High Water.
- *Diurnal Low Water Inequality* (DLQ) is the difference between Mean Low Water and Mean Lower Low Water.
- *Great Diurnal Range* (Gt) is the difference between Mean Higher High Water and Mean Lower Low Water.

All of these tidal datums and differences have users that need a specific datum or difference for their particular use. The important point for users is to know which tidal datum their data are referenced to. Like geodetic vertical datums, local tidal datums are all different from one another, but they can be related to each other. The relationship of a local tidal datum (941 4290, San Francisco, California) to geodetic datums is illustrated in Table I.2.

Please note that in this example, NAVD 88 heights, which are the official national geodetic vertical control values, and LMSL heights, which are the official national local mean sea level values, at the San Francisco tidal station differ by almost one meter. Therefore, if a user obtained a set of heights relative to the local mean sea level and a second set referenced to NAVD 88, the two sets would disagree by about one meter due to the datum difference. In addition, the difference between MHW and MLLW is more than 1.5 m (five feet). Due to regulations and laws, some users relate their data to MHW, while others relate their data to MLLW. As long as a user knows which datum the data are referenced to, the data can be converted to a common reference and the data sets can be combined.

Summary

This is the first in a four-part series of papers that will review the fundamental concept of height. The National Geodetic Survey will not, in the future, create or maintain elevation benchmarks by leveling. Instead, NGS will assign vertical control by estimating orthometric heights from ellipsoid heights as computed from GPS measurements. This marks a significant shift in how the United States' vertical control is created and maintained. Furthermore, practicing surveyors and mappers who use GPS are now confronted with using ellipsoid heights in their everyday work, something that was practically unheard of before GPS. The relationship between ellipsoid heights and orthometric heights is not simple, and it is the purpose of this series of papers to examine that relationship.

PBM 180 1946	-----	5.794 m (the Primary Bench Mark)
Highest Water Level	-----	4.462 m
MHHW	-----	3.536 m
MHW	-----	3.353 m
MTL	-----	2.728 m
MSL	-----	2.713 m
DTL	-----	2.646 m
NGVD 1929	-----	2.624 m
MLW	-----	2.103 m
NAVD 88	-----	1.802 m
MLLW	-----	1.759 m
Lowest Water Level	-----	0.945 m

Table I.2. Various tidal datums and vertical datums for PBM 180 1946.

This first paper reviewed reference ellipsoids and mean sea level datums. Reference ellipsoids are models of the Earth's shape and fall into two distinct categories: local and equipotential. Local reference ellipsoids were created by continental-sized triangulation networks and were employed as a computational surface but not as a vertical datum in the ordinary sense. Local reference ellipsoids are geometric in nature; their size and shape were determined by purely geometrical means. They were also custom-fit to a particular locale due to the impossibility of observing stations separated by oceans. Equipotential ellipsoids include the geometric considerations of local reference ellipsoids, but they also include information about the Earth's mass and rotation. They model the mean sea level equipotential surface that would result from both the redistribution of the Earth's mass caused by its rotation, as well as the centripetal effect of the rotation. It is purely a mathematical construct derived from observed physical parameters of the Earth. Unlike local reference ellipsoids, equipotential ellipsoids are routinely used as a vertical datum. Indeed, all heights directly derived from GPS measurements are ellipsoid heights.

Even though equipotential ellipsoids are used as vertical datums, most practicing surveyors and mappers use orthometric heights, not ellipsoid heights. The first national mean sea level datum in the United States was the NGVD 29. NGVD 29 heights were assigned to fiducial benchmarks through a least-squares adjustment of local height networks tied to separate tide gauges around the nation. It was observed at that time that mean sea level was inconsistent through these stations on the order of meters, but the error was blurred through the network statistically. The most recent general adjustment of the U.S. network, which is known as NAVD 88, was completed in June 1991. Only a single tide gauge was held fixed in NAVD 88 and, consequently, the inconsistencies between tide gauges were not distributed through the network adjustment, but there will be a bias at each mean sea level station between NAVD 88 level surface and mean sea level.

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